Language: **English**

Day: **1**

Thursday, April 12, 2012

Problem 1. Let ABC be a triangle with circumcentre O. The points D, E and F lie in the interiors of the sides BC, CA and AB respectively, such that DE is perpendicular to CO and DF is perpendicular to BO. (By *interior* we mean, for example, that the point D lies on the line BC and D is between B and C on that line.)

Let K be the circumcentre of triangle AFE. Prove that the lines DK and BC are perpendicular.

Problem 2. Let n be a positive integer. Find the greatest possible integer m, in terms of n, with the following property: a table with m rows and n columns can be filled with real numbers in such a manner that for any two different rows $[a_1, a_2, \ldots, a_n]$ and $[b_1, b_2, \ldots, b_n]$ the following holds:

$$\max(|a_1 - b_1|, |a_2 - b_2|, \ldots, |a_n - b_n|) = 1.$$

Problem 3. Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that

$$f\big(\, yf(x+y) + f(x) \,\big) = 4x + 2yf(x+y)$$

for all $x, y \in \mathbb{R}$.

Problem 4. A set A of integers is called *sum-full* if $A \subseteq A + A$, i.e. each element $a \in A$ is the sum of some pair of (not necessarily different) elements $b, c \in A$. A set A of integers is said to be *zero-sum-free* if 0 is the only integer that cannot be expressed as the sum of the elements of a finite nonempty subset of A.

Does there exist a sum-full zero-sum-free set of integers?

Time: 4 hours and 30 minutes
Each problem is worth 7 points