EGMO 2022 Hungary Eger

European Girls' Mathematical Olympiad

Language: **English**

Day: **1**

Friday, April 8, 2022

Problem 1. Let ABC be an acute-angled triangle in which $BC < AB$ and $BC < CA$. Let point P lie on segment AB and point Q lie on segment AC such that $P \neq B$, $Q \neq C$ and $BQ = BC = CP$. Let T be the circumcentre of triangle APQ, H the orthocentre of triangle ABC, and S the point of intersection of the lines BQ and CP. Prove that T, H and S are collinear.

Problem 2. Let $\mathbb{N} = \{1, 2, 3, \ldots\}$ be the set of all positive integers. Find all functions $f : \mathbb{N} \to \mathbb{N}$ such that for any positive integers a and b, the following two conditions hold:

(1) $f(ab) = f(a)f(b)$, and

(2) at least two of the numbers $f(a)$, $f(b)$ and $f(a + b)$ are equal.

Problem 3. An infinite sequence of positive integers $a_1, a_2, \ldots$ is called *good* if

(1) a_1 is a perfect square, and

(2) for any integer $n \geq 2$, a_n is the smallest positive integer such that

$$na_1 + (n-1)a_2 + \ldots + 2a_{n-1} + a_n$$

is a perfect square.

Prove that for any good sequence $a_1, a_2, \ldots$, there exists a positive integer k such that $a_n = a_k$ for all integers $n \geq k$.

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Time: 4 hours and 30 minutes
Each problem is worth 7 points

To make this a fair and enjoyable contest for everyone, please do not mention or refer to the problems on the internet or on social media until Saturday 9 April, 22:00 UTC (15:00 Pacific Daylight Time, 00:00 (Sunday) Central European Summer Time, 08:00 (Sunday) Australian Eastern Standard Time).