EGMO 2022 Hungary Eger
European Girls' Mathematical Olympiad

Language: **English**

Day: **2**

Saturday, April 9, 2022

Problem 4. Given a positive integer $n \geq 2$, determine the largest positive integer N for which there exist $N+1$ real numbers $a_0, a_1, \ldots, a_N$ such that

(1) $a_0 + a_1 = -\frac{1}{n}$, and

(2) $(a_k + a_{k-1})(a_k + a_{k+1}) = a_{k-1} - a_{k+1}$ for $1 \leq k \leq N-1$.

Problem 5. For all positive integers n, k, let $f(n, 2k)$ be the number of ways an $n \times 2k$ board can be fully covered by nk dominoes of size 2×1. (For example, $f(2,2) = 2$ and $f(3,2) = 3$.)
Find all positive integers n such that for every positive integer k, the number $f(n, 2k)$ is odd.

Problem 6. Let $ABCD$ be a cyclic quadrilateral with circumcentre O. Let the internal angle bisectors at A and B meet at X, the internal angle bisectors at B and C meet at Y, the internal angle bisectors at C and D meet at Z, and the internal angle bisectors at D and A meet at W. Further, let AC and BD meet at P. Suppose that the points X, Y, Z, W, O and P are distinct.
Prove that O, X, Y, Z and W lie on the same circle if and only if P, X, Y, Z and W lie on the same circle.

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Time: 4 hours and 30 minutes
Each problem is worth 7 points

To make this a fair and enjoyable contest for everyone, please do not mention or refer to the problems on the internet or on social media until Saturday 9 April, 22:00 UTC (15:00 Pacific Daylight Time, 00:00 (Sunday) Central European Summer Time, 08:00 (Sunday) Australian Eastern Standard Time).