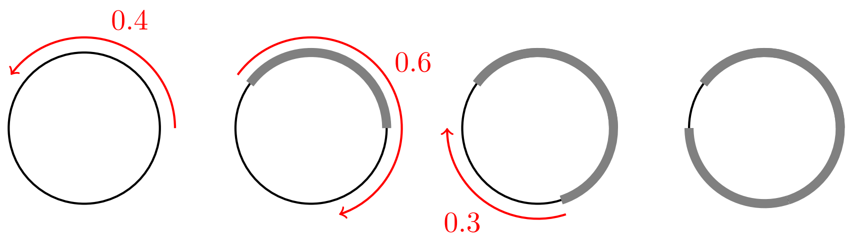
Language: **English**

Day: **2**

Sunday, April 16, 2023

Problem 4. Turbo the snail sits on a point on a circle with circumference 1. Given an infinite sequence of positive real numbers $c_1, c_2, c_3, \ldots$, Turbo successively crawls distances $c_1, c_2, c_3, \ldots$ around the circle, each time choosing to crawl either clockwise or counterclockwise.

For example, if the sequence $c_1, c_2, c_3, \ldots$ is $0.4, 0.6, 0.3, \ldots$, then Turbo may start crawling as follows:



Determine the largest constant $C > 0$ with the following property: for every sequence of positive real numbers $c_1, c_2, c_3, \ldots$ with $c_i < C$ for all i, Turbo can (after studying the sequence) ensure that there is some point on the circle that it will never visit or crawl across.

Problem 5. We are given a positive integer $s \geqslant 2$. For each positive integer k, we define its *twist* k' as follows: write k as $as + b$, where a, b are non-negative integers and $b < s$, then $k' = bs + a$. For the positive integer n, consider the infinite sequence $d_1, d_2, \ldots$ where $d_1 = n$ and d_{i+1} is the twist of d_i for each positive integer i.

Prove that this sequence contains 1 if and only if the remainder when n is divided by $s^2 - 1$ is either 1 or s.

Problem 6. Let ABC be a triangle with circumcircle Ω. Let S_b and S_c respectively denote the midpoints of the arcs AC and AB that do not contain the third vertex. Let N_a denote the midpoint of arc BAC (the arc BC containing A). Let I be the incentre of ABC. Let ω_b be the circle that is tangent to AB and internally tangent to Ω at S_b, and let ω_c be the circle that is tangent to AC and internally tangent to Ω at S_c. Show that the line IN_a, and the line through the intersections of ω_b and ω_c, meet on Ω.

The incentre of a triangle is the centre of its incircle, the circle inside the triangle that is tangent to all three sides.

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Time: 4 hours and 30 minutes
Each problem is worth 7 points

The problems are confidential until Sunday 16 April, 22:00 UTC (00:00 (Monday) Central European Summer Time).