Tuesday, April 9, 2019

Problem 1. Find all triples (a, b, c) of real numbers such that $ab + bc + ca = 1$ and

$$a^2b + c = b^2c + a = c^2a + b.$$

Problem 2. Let n be a positive integer. Dominoes are placed on a $2n \times 2n$ board in such a way that every cell of the board is adjacent to exactly one cell covered by a domino. For each n, determine the largest number of dominoes that can be placed in this way.

(A *domino* is a tile of size 2×1 or 1×2. Dominoes are placed on the board in such a way that each domino covers exactly two cells of the board, and dominoes do not overlap. Two cells are said to be *adjacent* if they are different and share a common side.)

Problem 3. Let ABC be a triangle such that $\angle CAB > \angle ABC$, and let I be its incentre. Let D be the point on segment BC such that $\angle CAD = \angle ABC$. Let ω be the circle tangent to AC at A and passing through I. Let X be the second point of intersection of ω and the circumcircle of ABC. Prove that the angle bisectors of $\angle DAB$ and $\angle CXB$ intersect at a point on line BC.

Time: 4 hours and 30 minutes
Each problem is worth 7 points