

**7th Middle European Mathematical Olympiad**

TEAM COMPETITION

25th August 2013**Problem T-1.** Find all functions $f: \mathbb{R} \rightarrow \mathbb{R}$ such that

$$f(xf(x) + 2y) = f(x^2) + f(y) + x + y - 1$$

for all $x, y \in \mathbb{R}$.**Problem T-2.** Let $x, y, z, w \in \mathbb{R} \setminus \{0\}$ such that $x + y \neq 0$, $z + w \neq 0$, and $xy + zw \geq 0$. Prove the inequality

$$\left(\frac{x+y}{z+w} + \frac{z+w}{x+y}\right)^{-1} + \frac{1}{2} \geq \left(\frac{x}{z} + \frac{z}{x}\right)^{-1} + \left(\frac{y}{w} + \frac{w}{y}\right)^{-1}.$$

Problem T-3. There are $n \geq 2$ houses on the northern side of a street. Going from the west to the east, the houses are numbered from 1 to n . The number of each house is shown on a plate. One day the inhabitants of the street make fun of the postman by shuffling their number plates in the following way: for each pair of neighbouring houses, the current number plates are swapped exactly once during the day.

How many different sequences of number plates can they have at the end of the day?

Problem T-4. Consider finitely many points in the plane with no three points on a line. All these points can be coloured red or green such that any triangle with vertices of the same colour contains a point of the other colour in its interior.

What is the maximal possible number of points with this property?

Problem T-5. Let ABC be an acute triangle. Construct a triangle PQR such that $AB = 2PQ$, $BC = 2QR$, $CA = 2RP$, and the lines PQ , QR , and RP pass through the points A , B , and C , respectively. (All six points A , B , C , P , Q , and R are distinct.)**Problem T-6.** Let ABC be an acute triangle, let K point inside a triangle ABC , so that BC is common tangent of circumcircles of AKB , AKC and let $D = CK \cap AB$, $E = BK \cap AC$. Let F be an intersection of BC and axis line of DE . Let P , Q are intersection points of circle k with center F and radius FD and circumcircle of ABC . Prove that PQ is diameter of k .**Problem T-7.** The numbers from 1 to 2013^2 are written row by row into a table consisting of 2013×2013 cells. Afterwards, all columns and all rows containing at least one of the perfect squares $1, 4, 9, \dots, 2013^2$ are deleted.

How many cells remain?

Problem T-8. The expression

$$\pm \square \pm \square \pm \square \pm \square \pm \square \pm \square$$



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is written on the blackboard. Two players, A and B , play a game, taking turns. Player A takes the first turn. In each turn, the player on turn replaces a symbol \square by a positive integer. After all the symbols \square are replaced, player A replaces each of the signs \pm by either $+$ or $-$, independently of each other. Player A wins if the value of the expression on the blackboard is not divisible by any of the numbers $11, 12, \dots, 18$. Otherwise, player B wins.

Determine which player has a winning strategy.

Time: 5 hours

Time for questions: 60 min

Each problem is worth 8 points.

The order of the problems does not depend on their difficulty.