

**Problem I-1**

Let  $\mathbb{Q}^+$  denote the set of all positive rational numbers and let  $\alpha \in \mathbb{Q}^+$ . Determine all functions  $f: \mathbb{Q}^+ \rightarrow (\alpha, +\infty)$  satisfying

$$f\left(\frac{x+y}{\alpha}\right) = \frac{f(x) + f(y)}{\alpha}, \quad \text{for all } x, y \in \mathbb{Q}^+.$$

**Problem I-2**

The two figures depicted below consisting of 6 and 10 unit squares, respectively, are called *staircases*.



Consider a  $2018 \times 2018$  board consisting of  $2018^2$  cells, each being a unit square. Two arbitrary cells were removed from the same row of the board. Prove that the rest of the board cannot be cut (along the cell borders) into staircases (possibly rotated).

**Problem I-3**

Let  $ABC$  be an acute-angled triangle with  $AB < AC$ , and let  $D$  be the foot of its altitude from  $A$ . Let  $R$  and  $Q$  be the centroids of the triangles  $ABD$  and  $ACD$ , respectively. Let  $P$  be a point on the line segment  $BC$  such that  $P \neq D$  and the points  $P, Q, R$  and  $D$  are concyclic. Prove that the lines  $AP, BQ$  and  $CR$  are concurrent.

**Problem I-4**

(a) Prove that for every positive integer  $m$  there exists an integer  $n \geq m$  such that

$$\left\lfloor \frac{n}{1} \right\rfloor \cdot \left\lfloor \frac{n}{2} \right\rfloor \cdots \left\lfloor \frac{n}{m} \right\rfloor = \binom{n}{m}. \quad (*)$$

(b) Denote by  $p(m)$  the smallest integer  $n \geq m$  such that the equation (\*) holds. Prove that  $p(2018) = p(2019)$ .

*Remark: For a real number  $x$ , we denote by  $\lfloor x \rfloor$  the largest integer not larger than  $x$ .*